KD Trees

Learning Objectives

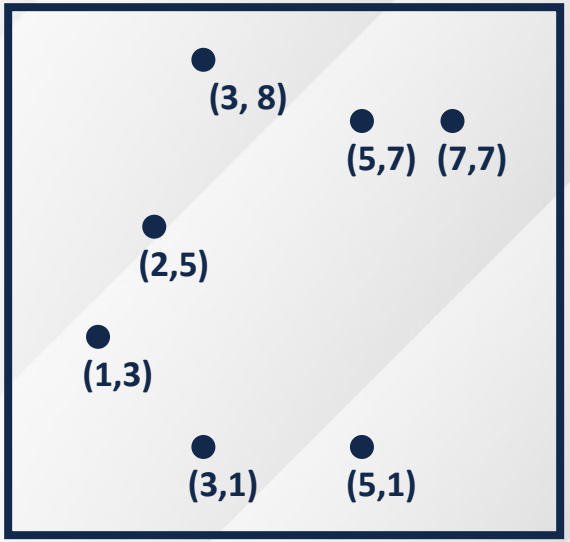
1. Understand what a KD-Tree is
2. Understand the KD-Tree Construction Algorithm

2D Range Based Searches

Consider points in 2D: $\mathbf{p = \{p_1, p_2, \ldots, p_n\}}$

Q: What points are in the rectangle:
[(4, 4), (6, 9)]?

Q: What is the nearest point to **(4,3)**?

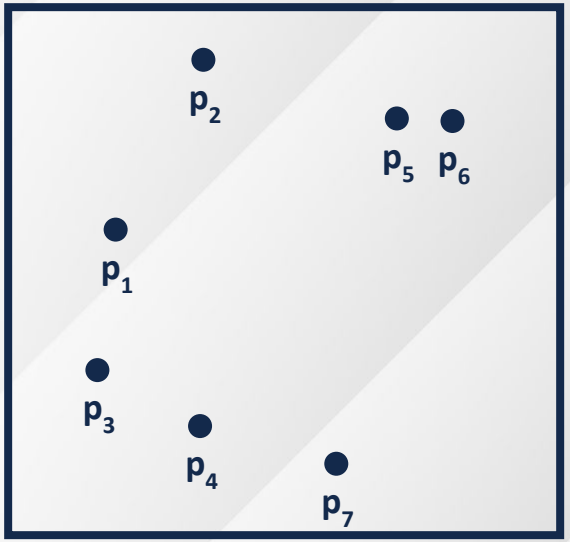


2D Range Based Searches

Consider points in 2D: $\mathbf{p = \{p_1, p_2, \ldots, p_n\}}$

Tree Construction:

1. Find median point along a dimension
2. Partition nodes
3. Go to next dimension
4. Recursively build left subtree
5. Recursively build right subtree



Quick Select Algorithm

Partitions elements in average linear time

Doesn’t sort

Ex. [11, 6, 44, 41, 33, 57, 2]

For a random pivot index, k = 3

[11, 6, 44, 2, 33, 57, 41], i = 0, small = 0

Quick Select Algorithm

Partitions elements in average linear time

Doesn’t sort

Ex. [11, 6, 44, 41, 33, 57, 2]

For a random pivot index, k = 3

[11, 6, 44, 2, 33, 57, 41], i = 0, small = 0

[11, 6, 44, 2, 33, 57, 41], i = 1, small = 1

Quick Select Algorithm

Partitions elements in average linear time

Doesn’t sort

Ex. [11, 6, 44, 41, 33, 57, 2]

For a random pivot index, k = 3

[11, 6, 44, 2, 33, 57, 41], i = 0, small = 0

[11, 6, 44, 2, 33, 57, 41], i = 1, small = 1

[11, 6, 44, 2, 33, 57, 41], i = 2, small = 2

Quick Select Algorithm

Partitions elements in average linear time

Doesn’t sort

Ex. [11, 6, 44, 41, 33, 57, 2]

For a random pivot index, k = 3

[11, 6, 44, 2, 33, 57, 41], i = 0, small = 0

[11, 6, 44, 2, 33, 57, 41], i = 1, small = 1

[11, 6, 44, 2, 33, 57, 41], i = 2, small = 2

[11, 6, 44, 2, 33, 57, 41], i = 3, small = 2

Quick Select Algorithm

[11, 6, 44, 2, 33, 57, 41], i = 3, small = 2

Quick Select Algorithm

[11, 6, 44, 2, 33, 57, 41], i = 3, small = 2

[11, 6, 2, 44, 33, 57, 41], i = 4, small = 3

Quick Select Algorithm

[11, 6, 44, 2, 33, 57, 41], i = 3, small = 2

[11, 6, 2, 44, 33, 57, 41], i = 4, small = 3

[11, 6, 2, 33, 44, 57, 41], i = 5, small = 4

Quick Select Algorithm

[11, 6, 44, 2, 33, 57, 41], i = 3, small = 2

[11, 6, 2, 44, 33, 57, 41], i = 4, small = 3

[11, 6, 2, 33, 44, 57, 41], i = 5, small = 4

[11, 6, 2, 33, 44, 57, 41], i = 6, small = 4

Quick Select Algorithm

[11, 6, 44, 2, 33, 57, 41], i = 3, small = 2

[11, 6, 2, 44, 33, 57, 41], i = 4, small = 3

[11, 6, 2, 33, 44, 57, 41], i = 5, small = 4

[11, 6, 2, 33, 44, 57, 41], i = 6, small = 4

[11, 6, 2, 33, 41, 57, 44]

Quick Select Algorithm

[11, 6, 44, 2, 33, 57, 41], i = 3, small = 2

[11, 6, 2, 44, 33, 57, 41], i = 4, small = 3

[11, 6, 2, 33, 44, 57, 41], i = 5, small = 4

[11, 6, 2, 33, 44, 57, 41], i = 6, small = 4

[11, 6, 2, 33, 41, 57, 44]

Recurse until you pivot around the median